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ANALYTICAL INVESTIGATION OF THE NONLINEAR CHARACTERISTICS OF A RECTANGULAR WING OF SMALL ASPECT RATIO

by

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6) ANALYTICAL INVESTIGATION OF THE NONLINEAR CHARACTERISTICS OF A RECTANGULAR WING OF SMALL ASPECT RATIO

(ANALITICHESKOE ISSLEDOVANIE NELINEINYKH KHARAKTERISTIK PRYAMOUGOL'NOGO KRYLA MALOGO UDLINENIYA)

10) by

V. F. Molchanov

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AUTHOR'S SUMMARY

In Ref 1, the form was found of the main nonlinear term of the expansion of the lift coefficient for a rectangular wing. In this paper, the form is found of all terms of the expansion of the lift and moment coefficients. Generalisations are given for the case of certain non-steady flows. Results of calculations are presented.

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1 Nonlinear effects of forces of an ideal medium acting on a wing are, in general, governed by the presence of a surface of discontinuity of the tangential components of the velocity. Within an accuracy of the order of α^2, these effects can be found by the method of plane sections. Here α is the angle of incidence, λ is the aspect ratio. α varies within the limits:

$$0 \leqslant \alpha \leqslant \alpha'\lambda \quad , \tag{1-1}$$

where α' is a constant.

By virtue of (1-1), $\alpha\lambda^2 \sim \alpha^2$, which makes it possible, in calculation, to replace $\sin\alpha$ by α. The problem concerning the movement of a rectangular wing comes down to one of finding a plane, non-steady separated flow of an ideal liquid about a plate instantaneously brought into motion along its normal, with a velocity v_n. Thereupon, the following relationship arises:

$$\alpha V_\infty = v_n \ , \qquad tV_\infty = h \ , \tag{1-2}$$

where v_n is the velocity of motion of the plate; h is the distance between the leading edge of the wing and the plane of the section of the wing normal to the flow; V_∞ is the velocity at infinity; t is the time elapsed from the moment of the start of the motion of particles of the medium in the given section.

The case of non-steady movement of a rectangular wing also reduces to this plane problem when the wing is momentarily brought into motion with a constant velocity V_∞.

If t_0 is the time elapsed from the start of the wing motion, and h_0 is the chord of the wing, then:

$$t = \begin{cases} \dfrac{h}{V_\infty} & \text{for } 0 \leqslant h \leqslant V_\infty t_0 \\[2ex] t_0 & \text{for } V_\infty t_0 < h \leqslant h_0 \end{cases} \quad . \tag{1-3}$$

Then, as before, t is the time elapsed from the start of movement of particles of the medium in the given section.

If the wing is placed in a shock tube, and if t_0 is the time elapsed from the start of movement of the particles of the medium near the leading edge of the wing, then for the sections:

$$0 \leqslant h \leqslant V_\infty t_0 \ , \qquad t = \frac{h}{V_\infty} \ , \tag{1-4}$$

while, in sections $V_\infty t_0 < h \leqslant h_0$, motion is absent. In fact, in the vicinity of $h = V_\infty t_0$, there is a transition zone. However, its rejection leads to an error of the same order, $\alpha\lambda^2$.

The solution of the two-dimensional problem makes it possible to find the drag force on the plate $Y(t)$. This force is numerically equal to the normal force acting on a small part of the wing included between sections h and $h + \Delta h$, relative to the distance between the sections Δh. Therefore, the following expressions are obtained for the lift coefficient c_y and for the moment coefficient M_z, relative to the leading edge.

$$c_y = \frac{2}{\rho v^2 S} \int_0^{h_0} Y dh , \qquad M_z = -\frac{2}{\rho v^2 S} \int_0^{h_0} hY dh , \tag{1-5}$$

where Y must be expressed in terms of h, V_∞ and t_0.

In practice, it is more convenient to use the impulse $D(t)$ instead of the force $Y(t)$:

$$D(\ ) = \int_0^{t_0} Y dt . \tag{1-6}$$

Thus the problem of investigation of the characteristics of the wing c_y and M_z reduces to the determination of the impulse $D(t)$ as a function of the time, for the two-dimensional case of non-steady separated flow of a plane flow of an ideal liquid around a plate.

2 If the surfaces of tangential discontinuity formed near the edge of the plate are known, then the velocity field can be found according to known formulae. Therefore, the problem of finding $D(t)$ comes down to finding these surfaces.

The equations of motion for surfaces of tangential discontinuity can be written in the following form:

$$U(\Gamma,t) + \langle V(\Gamma,t)\rangle = \frac{dw^*}{dt} \tag{2-1}$$

$$\left.\begin{aligned} W(\Gamma,t) , \quad \Gamma_0 \geqslant \Gamma \geqslant 0 , \quad \Gamma_0(t) \\ \operatorname{Im}\{U(\Gamma_0,t) + \langle V(\Gamma_0,t)\rangle\} = 0 \end{aligned}\right\} . \tag{2-2}$$

Here W is the point of the complex plane lying at the surface of the discontinuity; Γ is the potential jump at point W; Γ_0 is the potential jump at the edge of the plate; U is the complex velocity of the non-separated flow of the stream v_n/i about the plate, directed towards the plate along the normal; V is the complex velocity induced by discontinuities in the presence of the plate (discontinuities adjoin its edges); $\langle V\rangle$ is the half-sum of the values V, calculated for both sides of the discontinuity at the point W. The superior asterisk denotes a complex-conjugate value.

The derivative in the right-hand part of (2-1) applies for the condition Γ = const. Equation (2-1) is obtained on the basis of the known properties of a tangential discontinuity, its impermeability and the absence of a pressure jump. Equation (2-2) is obtained on the basis of Zhukovskii's postulate. The systems (2-1) and (2-2) are complete in that they contain the functions $W(\Gamma,t)$ and $\Gamma_0(t)$, but they must be supplemented by a relationship between $W(\Gamma,t)$ and the functions Γ and $\langle V \rangle$. This relationship is conveniently found by introducing the auxiliary complex plane z. Without destroying the generality, it is possible to consider the half-width of the wing as being equal to unity, and the edges as lying at points $i+$ and $-i$ in the complex plane W. Then z is defined by the function:

$$W = \sqrt{1 + z^2} \quad , \tag{2-3}$$

mapping the exterior of the segment $(-1, +1)$ of the plane z on the exterior of the segment $(-i, +i)$ of the plane W.

The relationship between $W(\Gamma,t)$ and Γ, $W(\Gamma,t)$ and $\langle V \rangle$ is given by the following expressions:

$$\Gamma = u \frac{dz}{dW} , \qquad \langle V \rangle = \langle v \rangle \frac{dz}{dW} , \tag{2-4}$$

$$u = \frac{v_n}{i} , \qquad \langle v \rangle = \frac{1}{2\pi i} \int_0^{\Gamma_0} \frac{d\Gamma}{z - z_*} - \frac{1}{2\pi i} \int_0^{\Gamma_0} \frac{d\Gamma}{z - z_*^*} . \tag{2-5}$$

Here the inferior asterisk indicates a dependence of the given function on the variable of integration.

The expression for $D(t)$ is obtained on the basis of formulae in Ref 2:

$$D = \pi v_n + \Delta D \quad , \tag{2-6}$$

$$\Delta D = 2\rho \mathrm{Re} \int_0^{\Gamma_0} z_* d\Gamma \quad . \tag{2-7}$$

Let us change to new variables. Let us suppose that:

$$s = v_n t , \qquad \Gamma_0 = v_n \gamma , \qquad \frac{\Gamma}{\Gamma_0} = 1 - \omega \qquad 0 \leqslant \omega \leqslant 1 . \tag{2-8}$$

Moreover, in order to abbreviate the writing of functions, Γ and z will be given a supplementary definition for the case of negative values of ω, in accordance with the following expressions:

$$\Gamma(-\omega) = \Gamma(\omega) , \qquad z(-\omega) = -z^*(\omega) \quad . \tag{2-9}$$

In terms of these variables, the equations of motion take the following form:

$$\left.\begin{aligned} &\{p(0) - p(z)\}E + (1 - \omega)\frac{\partial z}{\partial \omega}\frac{\partial p}{\partial s} - p(0)\frac{\partial z}{\partial s} = 0 , \\ &p(z) = \int_{-1}^{+1} \frac{\delta_\star d\omega}{z - z_\star} , \qquad p(0) = \int_{-1}^{+1} \frac{\delta_\star d\omega}{-z_\star} , \qquad E = \frac{1}{i}\left|\frac{1 + z^2}{z^2}\right| \end{aligned}\right\} \tag{2-10}$$

$$2\tau = \gamma p(0) , \tag{2-11}$$

$$\Delta D = \rho v_n \gamma \mathrm{Re} \int_{-1}^{+1} z_\star d\omega , \tag{2-12}$$

where $\delta_\star = \delta(\omega) = 1$ for $\omega > 0$ and -1 for $\omega < 0$. The problem reduces to the solution of system (2-10) and the introduction of results in (2-11) and (2-12).

3 Let us investigate the principle of finding the solution.

Let the solution of a certain physical problem reduce to the solution of the operator equation:

$$Fz = 0 , \qquad z = z(\omega, s) , \tag{3-1}$$

which, by assumption, has a unique solution, and this solution can be represented in the following way:

$$z = \sum_{n=0}^{\infty} a_n \phi_n, \quad a_n(\omega), \quad \phi_n(s) \tag{3-2}$$

(Editor's note: a minor correction has been made in (3-5).)

$$\phi_n \not\equiv 0 , \tag{3-3}$$

$$\phi_n \to 0 \qquad \text{for} \qquad s \to 0 , \tag{3-4}$$

$$\frac{\phi_{n+1}}{\phi_n} \to 0 \qquad \text{for} \qquad s \to 0 , \qquad n = 0, 1, 2, \ldots . \tag{3-5}$$

It is necessary to find the functions $\phi_n(s)$. In the general case, for this purpose equation (3-1) must be solved. However, if the operator F has certain special properties and if, from physical considerations, one can obtain additional restrictions on the desired solution z , then the function ϕ_n can be found even without the direct solution of (3-1).

Let us consider the case when the operator F has the following three properties.

(1) For any function of z which can be represented as in (3-2) to (3-5) and which belongs to a certain acceptable region of definition, the expression Fz can be written in the following form:

$$Fz = \sum_{n=0}^{\infty} (A_n a)(\Phi_n \phi) \quad ; \tag{3-6}$$

here a and ϕ are infinitely-dimensioned vectors with the coordinates a_n and ϕ_n respectively; A_n and Φ_n are certain operators acting on a and ϕ . We observe that, using only the condition (3-5), it is not possible to place the expressions $\Phi_n \phi$ as a function of s in increasing order. It is not, for example, possible to compare the orders of the functions ϕ_0 and $\phi_0 d\phi_0/ds$. However, in expression (3-6) there are also terms which can be arranged in an order relationship established on the basis of conditions (3-5).

(2) If, using conditions (3-5), any term of (3-6) having a higher order than some other term in the latter expression is rejected, then only a finite number of terms depending solely on a_0 and ϕ_0 remain.

For the sake of simplicity, let us consider that only two terms remain. Let us call this residue the dominant part (DP):

$$DP(Fz) = (A_1 a_0)(\Phi_1 \phi_0) + (A_2 a_0)(\Phi_2 \phi_0) \quad . \tag{3-7}$$

(Translator's note: a minor correction has been made to (3-7).)

(3) The dominant part of the increment $Fz - F(z - z_k)$, where:

$$z_k = \sum_{n=k}^{\infty} a_n \phi_n \quad , \tag{3-8}$$

also consists of a finite number of terms, each of which depends only on a_0 , a_k , ϕ_0 , ϕ_k . Here the dependence on a_k , ϕ_k is linear, and the operators determining this dependence are independent of k . Accordingly, we shall investigate the case when the dominant part of the increment consists only of two terms:

$$DP\left[Fz - F(z - z_k)\right] = \left(A_1'(a_0)a_k\right)\left(\Phi_1'(\phi_0)\phi_k\right) + \left(A_2'(a_0)a_k\right)\left(\Phi_2'(\phi_0)\phi_k\right) \quad . \tag{3-9}$$

Depending on the vector ϕ , in the expression (3-7), the dominant term will be either the first, or the second, or both terms will have the same orders. The expression (3-6) may similarly equal zero only in the case when the sum of each one of the dominant terms is separately equal to zero. Therefore, a_0 can satisfy one of the following equations:

$$A_1 a_0 = 0 \quad , \tag{3-10}$$

$$A_2 a_0 = 0 \quad , \tag{3-11}$$

$$\left.\begin{aligned} (A_1 a_0) c_1^0 + A_2 a_0 &= 0 \quad , \\ (\varphi_1 \varphi_0)/(\varphi_2 \varphi_0) &= c_1^0 \quad , \\ c_1^0 &= \text{const.} \end{aligned}\right\} \tag{3-12}$$

It is possible to give preference to one of the equations (3-10), (3-11) or (3-12) on the basis of additional restrictions on the solution z obtained from physical considerations. In the given case, the condition of a unique solution is sufficient to justify rejection of equations (3-10) and (3-11), since they do not determine the function φ_0 .

In the third case, however, a unique solution is possible.

If (3-7) were to consist of n terms, then it would be necessary to investigate $2^n - 1$ equations. Functions a_1 , φ_1 are found in a similar manner. It is sufficient merely to carry out a substitution of the variables $z = a_0\varphi_0 + z_1$, where z_1 is determined by the sum (3-8), and to require that the dominant member of the expression $F(a_0\varphi_0 + z_1) = 0$. Thus:

$$F(a_0\varphi_0 + z_1) = F(a_0\varphi_0) + \{F(a_0\varphi_0 + z_1) - F(a_0\varphi_0)\} \tag{3-13}$$

and, as soon as a_0 and φ_0 are known, then the dominant member (DM) of the first term can be picked out directly. Moreover, by virtue of (3-12), its order is higher than zero. Taking condition (3-6) into consideration, let us suppose that:

$$DM\{F(a_0\varphi_0)\} = B_1(a_0)\psi_1(\varphi_0) \quad . \tag{3-14}$$

From the expressions contained in brackets in (3-13) we preferentially take out the dominant part. Taking (3-9) into consideration, we obtain:

$$DP\{F(a_0\varphi_0 + z_1) - F(a_0\varphi_0)\} = \{A_1'(a_0)a_1\}\{\varphi_1'(\varphi_0)\varphi_1\} + \{A_2'(a_0)a_1\}\{\varphi_2'(\varphi_0)\varphi_1\} \tag{3-15}$$

If $a_0\varphi_0$ is not the solution of problem (3-1), then $B_1(a_0) \not\equiv 0$ and consequently, the order of (3-14) cannot be less than the order of the terms in (3-15). On the other hand, none of the terms in (3-15) can have an order less than that of (3-14), since, in such a case, for a_1 the trivial solution $a_1 \equiv 0$ would be found. One of the following three systems of equations is possible for a_1 :

$$\left.\begin{aligned} B_1(a_0) + c_1^1\{A_1'(a_0)a_1\} &= 0 \\ \{\varphi_1'(\varphi_0)\varphi_1\}\big/\psi_1(\varphi_0) = c_1^1 &= \text{const.} \end{aligned}\right\} \tag{3-16}$$

$$\left.\begin{aligned} B_1(a_0) + c_1^2\left(A_0'(a_0)a_1\right) &= 0 \\ \left(\phi_2'(\phi_0)\phi_1\right)\Big/\phi_1(\phi_0) &= c_1^2 = \text{const.} \end{aligned}\right\} \qquad (3\text{-}17)$$

$$\left.\begin{aligned} B_1(a_0) + c_1^1\left(A_1'(a_0)a_1\right) + c_1^2\left(A_2'(a_0)a_1\right) &= 0 \\ \left(\phi_1'(\phi_0)\phi_1\right)\Big/\phi_1(\phi_0) &= c_1^1 = \text{const.} \\ \left(\phi_2'(\phi_0)\phi_1\right)\Big/\phi_1(\phi_0) &= c_1^2 = \text{const.} \end{aligned}\right\} . \qquad (3\text{-}18)$$

System (3-16) or system *(3-17) can be rejected if its solution is such that the expressions* $\phi_1'(\phi_0)\phi_1$ and $\phi_2'(\phi_0)\phi_1$ have equal orders.

Let us suppose that all values of a_n , ϕ_n are found up to the number $n = k - 1$. We write:

$$\sum_{n=0}^{k-1} a_n \phi_n = z^{k-1} . \qquad (3\text{-}19)$$

Then, $z = z^{k-1} + z_k$. Repeating the calculation, we obtain:

$$\left.\begin{aligned} B_k(a^{k-1}) + c_k^1\left(A_1'(a_0)a_k\right) &= 0 \\ \left(\phi_1'(\phi_0)\phi_k\right)\Big/\phi_k(\phi^{k-1}) &= c_k^1 = \text{const.} \end{aligned}\right\} \qquad (3\text{-}20)$$

$$\left.\begin{aligned} B_k(a^{k-1}) + c_k^2\left(A_2'(a_0)a_k\right) &= 0 \\ \left(\phi_2'(\phi_0)\phi_k\right)\Big/\phi_k(\phi^{k-1}) &= c_k^2 = \text{const.} \end{aligned}\right\} \qquad (3\text{-}21)$$

$$\left.\begin{aligned} B_k(a^{k-1}) + c_k^1\left(A_1'(a_0)a_k\right) + c_k^2\left(A_2'(a_0)a_k\right) &= 0 \\ \left(\phi_1'(\phi_0)\phi_k\right)\Big/\phi_k(\phi^{k-1}) &= c_k^1 = \text{const.} \\ \left(\phi_2'(\phi_0)\phi_k\right)\Big/\phi_k(\phi^{k-1}) &= c_k^2 = \text{const.} \end{aligned}\right\} . \qquad (3\text{-}22)$$

where a^{k-1} and ϕ^{k-1} are vectors with the coordinates a_n , ϕ_n ; $0 \leqslant n \leqslant k - 1$;

$B_k(a^{k-1}).\phi_k(\phi^{k-1})$ is the dominant member of the expression Fz^{k-1} .

Systems (3-16) and (3-17) can be rejected on the basis of the same considerations.

[illegible]: The expansion (3-2) under the conditions of (3-5) and for given values of φ_n is unique[3], [illegible] if two functions coincide to the desired extent for the small range $0 \leqslant s \leqslant \varepsilon > 0$, then both they and their expansions coincide over the whole range of the determination.

It is sufficient, therefore, that the properties of sub-points (1) to (3) should be maintained merely for small values of s.

[illegible]: The properties of sub-points (2) and (3) are directly associated with the existence of the Fréchet derivative[4] of the operator F. If, for example:

$$F(z' + z'') = Fz' + F'(z')z'' + o(z'') , \tag{3-23}$$

where $F'(z')$ is a linear uniform operator depending on z (Fréchet derivative), then the properties of (3-7) and (2-8) are maintained if:

$$\lim \{F'(z_1')z''\} \Big/ \{F'(z_2')z''\} = 1 \qquad \text{for} \quad z_1'/z_2' \to 1, \; s \to 0 . \tag{3-24}$$

Thereupon the following formulae are valid:

$$DP\, Fz = DP\, F(a_0\varphi_0) , \tag{3-25}$$

$$DP\{Fz - F(z - z_k)\} \approx DP\{F'(a_0\varphi_0)a_k\varphi_k\} . \tag{3-26}$$

4 Applying the above considerations to equation (2-10), let us first of all formulate additional restrictions of a physical character:

(1) the absence of self-intersection of the tangential discontinuity:

$$|z(\omega_1,s) - z(\omega_2,s)| \Big/ |z(1,s)| \geqslant f(\omega_1,\omega_2) \tag{4-1}$$

(Editor's note: minor corrections have [illegible] (4-[illegible]) and ([illegible]).)

where $f(\omega_1,\omega_2)$ is a certain function;

$f(\omega_1,\omega_2) \neq 0 \quad$ for $\quad \omega_1 \neq \omega_2$;

(2) the boundedness of the velocity jump at the tangential discontinuity:

$$\left| \frac{\partial z(\omega,s)/\partial\omega}{z(1,s)} \right| > c' < 0 \tag{4-2}$$

where c' is a certain constant;

(3) the boundedness of the total circulation:

$$\gamma < c'' < \infty ; \tag{4-3}$$

(4) the tangential discontinuity springs from the edge:

$$z(0,s) = 0 ; \tag{4-4}$$

(5) moreover, we suppose that the series:

$$\sum_{n=0}^{\infty} \frac{da_n}{d\omega} \phi_n = \frac{\partial z}{\partial \omega} , \tag{4-5}$$

converges uniformly with respect to ω .

From (4-1), (4-2) and (4-5) it follows that for any values of ω_1 , ω_2 :

$$\lim \frac{z_1(\omega_1 s) - z_1(\omega_2 s)}{\phi_0 a_0(\omega_1) - \phi_0 a_0(\omega_2)} = 0 \qquad \text{for } s \to 0 . \tag{4-6}$$

Therefore, for small values of s :

$$\frac{1}{z(\omega_*, s) - z(\omega, s)} = \frac{1}{\phi_0\left(a_0(\omega_*) - a_0(\omega)\right)} \left(\frac{1}{1 + \dfrac{z_1(\omega_*, s) - z_1(\omega, s)}{\phi_0\left(a_0(\omega_*) - a_0(\omega)\right)}} \right)$$

$$= \frac{1}{\phi_0\left(a_0(\omega_*) - a_0(\omega)\right)} \left[1 + \sum_{k=1}^{\infty} \left(- \frac{z_1(\omega_*, s) - z_1(\omega, s)}{\phi_0\left(a_0(\omega^*) - a_0(\omega)\right)} \right)^k \right] . \tag{4-7}$$

This makes it possible to write $p(z)$ in the form (3-6). An analogous result is obtained for the operator E , and, consequently, for all the left-hand side of equation (2-10). The properties (3-7), (3-8) of the left-hand side of (2-10), written in the form (3-6) can be directly verified. Here formulae (3-25) and (3-26) may be used. Denoting by the symbol F the operator included in the left-hand side of (2-10), we obtain:

$$\mathrm{DP}\, Fz = \frac{1}{i\phi_0^3 |a_0^2|} \left(\int_{-1}^{+1} \frac{\delta_* d\omega}{-a_{0*}} - \int_{-1}^{+1} \frac{\delta_* d\omega}{a_0 - a_{0*}} \right) - \frac{\phi_0'}{\phi_0} \left((1 - \omega) \frac{da_0}{d\omega} + a_0 \right) \int_{-1}^{+1} \frac{\delta_* d\omega}{-a_{0*}} . \tag{4-8}$$

As in the case of point (3), we obtain three equations, two of which can be rejected not only on grounds of the conditions of uniqueness. For example, let us look at the equation:

$$\left((1 - \omega) \frac{da_0}{d\omega} + a_0 \right) \int_{-1}^{+1} \frac{\delta_* d\omega}{-a_{0*}} = 0 . \tag{4-9}$$

By virtue of (4-3) and (2-11),

$$\int_{-1}^{+1} \frac{\delta_* d\omega}{-a_{0*}} \neq 0 ,$$

therefore $(1 - \omega)\frac{d}{d\omega} a_0 + a_0 = 0$, $a_0 = c_1(1 - \omega)$, $c_1 = \text{const.}$;

which contradicts condition (4-4). *(Editor's note: more minor corrections here.)*

The second equation:

$$\int_{-1}^{+1} \frac{\delta_* d\omega}{-a_{0*}} - \int_{-1}^{+1} \frac{\delta_* d\omega}{a_0 - a_{0*}} = 0 \tag{4-10}$$

has the following solution:

$$a_0 = c_2 \sqrt{2\omega - \omega^2}$$

which does not satisfy condition (4-2), since $\partial a_0/\partial\omega = 0$ for $\omega = 1$. In this way, if a solution of the problem in the form (3-2) exists, then $a_0\phi_0$ satisfies the system (3-12). In so far as $\phi_1\phi_0 = \phi_0^{-3}$ *(editor's note: the original has ϕ_0^{δ})*, and $\phi_2\phi_0 = \phi_0'/\phi_0$, then, for ϕ_0 we obtain:

$$\phi_0'\phi_0^2 = c_1^0 , \qquad \phi_0 = \left(3c_1^0 s\right)^{\frac{1}{3}} , \tag{4-11}$$

where $\phi_0' = d\phi_0/ds$.

Since the constant factor associated with ϕ_0 is insignificant, then, for definiteness, it is possible to take $c_1^0 = \frac{1}{3}$, which by virtue of (3-12) unambiguously determines the equation for a_0 .

Transferring attention to the desired $\phi_1, \phi_2, \ldots, \phi_k$ we first obtain:

$$\begin{aligned} DP\left(Fz - F(z - z_k)\right) = {} & \frac{\phi_k}{\phi_0^4}\left[\frac{1}{i\left|a_0^2\right|}\left(\int_{-1}^{+1} \frac{a_{k*}\delta_*}{a_{0*}^2}\, d\omega + \int_{-1}^{+1} \frac{a_k - a_{k*}}{a_0 - a_{0*}}\, \delta_* d\omega\right)\right. \\ & \left. - \operatorname{Re}\left(\frac{2a_k}{a_0\left|a_0^2\right|}\right)\left(\int_{-1}^{+1} \frac{\delta_* d\omega}{-a_0} - \int_{-1}^{+1} \frac{\delta_* d\omega}{a_0 - a_{0*}}\right)\right] \\ & - \frac{\phi_k\phi_0'}{\phi_0^2}\left[\left(a_0 + 2(1 - \omega)\frac{da_0}{d\omega}\right)\int_{-1}^{+1} \frac{a_{k*}\delta_*}{a_{0*}^2}\, d\omega + (1 - \omega)\frac{da_k}{d\omega}\int_{-1}^{+1} \frac{\delta_* d\omega}{-a_{0*}}\right] \\ & + \frac{\phi_k'}{\phi_0}\left[(1 - \omega)\frac{da_0}{d\omega}\frac{a_{k*}\delta_*}{a_{0*}^2}\, d\omega - a_k\int_{-1}^{+1} \frac{\delta_* d\omega}{-a_{0*}}\right] . \end{aligned}$$

.....(4-12)

Since $\phi_0 = s^{\frac{1}{3}}$, then with an accuracy of up to the value of a constant multiplier of the function $\phi_0'\phi_0^2$ is equal to the function $1/\phi_0^4$. Therefore we have the situation described in point (3). Thus:

$$\Phi_1'(\phi_0)\phi_k = \phi_k s^{-\frac{4}{3}}, \qquad \Phi_2'(\phi_0)\phi_k = \phi_k' s^{-\frac{1}{3}}$$

Extracting the dominant member from the expression $F(\phi_0\phi_0)$, bearing in mind equation (3-12) and the function $\phi_0 = s^{\frac{1}{3}}$, we obtain:

$$\mathrm{DP}\ F\left(a_0 s^{\frac{1}{3}}\right) = -s^{-\frac{1}{3}}\left(\int_{-1}^{+1} \frac{\delta_* d\omega}{-a_{0*}} - \int_{-1}^{+1} \frac{\delta_* d\omega}{a_0 - a_{0*}}\right) i\mathrm{Re}\,\frac{a_0^2}{|a_0|} .$$

Thus, $\phi_1\phi_0 = s^{-\frac{1}{3}}$. Investigating the systems (3-16) to (3-18) according to principles laid down in point (3), the first two systems can be rejected. From the third system, with an accuracy of up to the value of a constant multiplier, we obtain:

$$\phi_1(s) = s . \tag{4-13}$$

Applying the method of mathematical induction for ϕ_k, we find:

$$\phi_k(s) = s^{(2k+1)/3} \qquad k = 0,1,2,\ldots . \tag{4-14}$$

Introducing the result found into (2-11):

$$\gamma = \sum_{n=0}^{\infty} \gamma_n s^{(2n+1)/3}, \qquad \gamma_n = \text{const.} \tag{4-15}$$

From (2-12), it follows that:

$$\Delta D = \rho v_n \sum_{n=1}^{\infty} s^{2n/3} D_n , \qquad D_n = \text{const.} \tag{4-16}$$

Using (4-16), (1-2), (1-5), (1-6) and (2-7), we obtain:

$$\left.\begin{aligned} c_y &= \alpha\lambda\pi/2 + \tfrac{1}{2}\alpha\lambda \sum_{n=1}^{\infty} D_n (2\alpha/\lambda)^{2n/3} , \\ M_z &= \alpha\lambda \sum_{n=1}^{\infty} \frac{2n}{2n+3} D_n (2\alpha/\lambda)^{2n/3} . \end{aligned}\right\} \tag{4-17}$$

For non-steady flows, we use (1-3) and (1-4); accordingly, we obtain c_y, M_z and c'_y, M''_z.

$$\left.\begin{aligned} c'_y &= \alpha\lambda\pi/2 + \tfrac{1}{2}\alpha\lambda \sum_{n=1}^{\infty} \left[\frac{2}{3}\, n\left(\frac{1}{\xi} - 1\right) + 1\right] D_n (2\alpha\xi/\lambda)^{2n/3} \\ M'_z &= \alpha\lambda \sum_{n=1}^{\infty} \left[\xi\, \frac{2n}{2n+3} + \frac{n}{6}\left(\frac{1}{\xi} - \xi\right)\right] D_n (2\alpha\xi/\lambda)^{2n/3} \end{aligned}\right\} \quad . \qquad (4\text{-}18)$$

$$\left.\begin{aligned} c''_y &= \alpha\lambda\pi/2 + \tfrac{1}{2}\alpha\lambda \sum_{n=1}^{\infty} D_n (2a\xi/\lambda)^{2n/3} \\ M''_z &= \alpha\lambda\xi \sum_{n=1}^{\infty} \frac{2n}{2n+3}\, D_n (2\alpha\xi/\lambda)^{2n/3} , \qquad \xi = t_0 V_\infty / h_0 \end{aligned}\right\} \quad . \qquad (4\text{-}19)$$

In order to find the coefficients D_n, it is necessary to solve the equations for a_n in systems (3-12) and (3-22). However, since, when one uses numerical methods with the application of the regularisation principle[5], there occurs a convergence not only in the case of the desired functions, but also in the case of all their derivatives, then the coefficients D_n can be found by means of a treatment of the results of the numerical calculation of the initial problem ((2-1) to (2-5)). In Fig 1 is shown the dependence, found by the numerical calculation of the initial problem, of $\Delta c_y \alpha^{-\frac{5}{3}} \lambda^{-\frac{1}{3}}$ as a function of $2\alpha\lambda$.

It is noticeable that, at point 0, there is a vertical tangent. This same quantity, as a function of $(2\alpha/\lambda)^{\frac{2}{3}}$ does not have a vertical tangent and, in the interval (0,1), it can, with a high degree of accuracy, be found approximately using the following polynomial (*cf* Fig 2):

$$\frac{\Delta c_y}{\alpha^{\frac{5}{3}} \lambda^{\frac{1}{3}}} = 3.61 + 1.20\beta - 0.55\beta^2 - 0.25\beta^3 , \qquad \beta = \left(\frac{2\alpha}{\lambda}\right)^{\frac{2}{3}} . \qquad (4\text{-}20)$$

The coefficients of this polynomial determine the first four coeficients D_n: $D_1 = 4.55$, $D_2 = 1.56$, $D_3 = -0.692$, $D_4 = 0.314$. The constant c_0 in the formula of A.A. Nikol'skii is numerically equal to the first coefficient of the polynomial (4-20).

A comparison of results of the calculation and of experiment is given in Ref 5.

In conclusion, two observations may be made.

Equation (4-10) corresponds to the case when, in the flow, the pressure is admitted of a point force applied to the end of the tangential discontinuity. Thereupon, the function ϕ_0 can be determined if one defines the law of change of this force.

By virtue of formulae (4-17), there is the following differential relationship between M_z and c_y:

$$\frac{\partial}{\partial \alpha} M_z = \alpha \frac{\partial}{\partial \alpha}\left(\frac{c_y}{\alpha}\right) .$$

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Figs 1&2

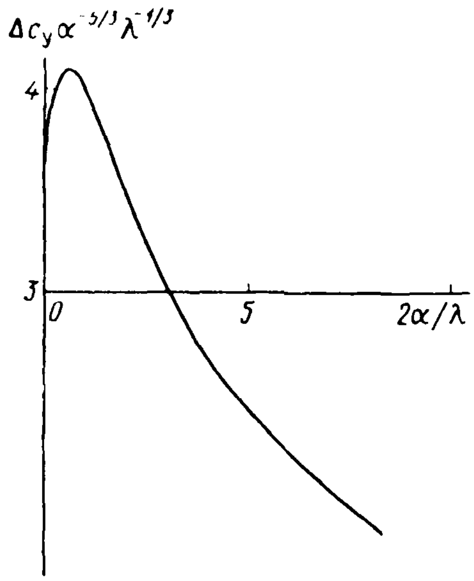
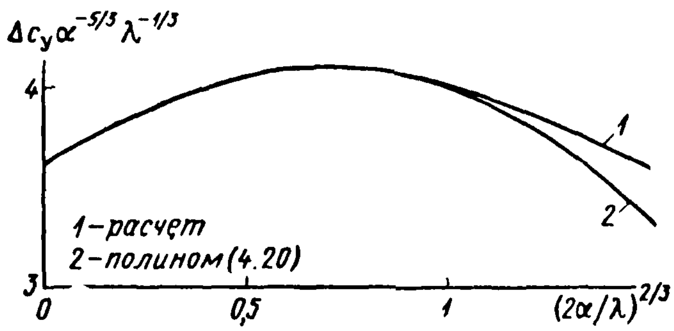


Fig 1



Key:
1 - calculation
2 - polynomial (equation (4-20))

Fig 2

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